Core Mathematics C4 For Edexcel Advanced Level

Paper K
Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration.

Full marks may be obtained for answers to ALL questions.

The booklet 'Mathematical Formulae and Statistical Tables', available from Edexcel, may be used.

When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner.
Answers without working may gain no credit.

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1. At time t seconds a sphere has radius r cm and volume $V\text{cm}^3$.

(a) Find $\dfrac{\mathrm{d}V}{\mathrm{d}r}$ in terms of r. **(1)**

(b) The radius is increasing at a rate of 0.1 cm s^{-1}, Find the rate at which the volume is increasing at the instant when the radius is 10 cm. Give your answer in terms of π. **(4)**

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2. (a) Find the gradient of the curve $x^2 + x \ln y + y = 10$ at the point (3, 1) **(3)**

(b) Find the x-coordinates of the stationary points on the curve

$$3x^2 + 2xy - 5y^2 + 16y = 0$$ **(5)**

3.

Figure 1

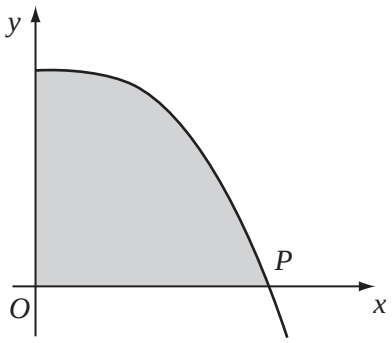

Figure 1 shows a sketch of part of the curve C whose parametric equations are

$$x = t^2, \qquad y = \cos t, \qquad t \geq 0$$

The curve crosses the positive x-axis for the first time at the point P.

(a) Find the coordinates of P. **(2)**

(b) (i) The shaded region bounded by the curve C and the coordinate axes has area A. Show that

$$A = \int_0^{\frac{\pi}{2}} 2t \cos t \ \mathrm{d}t.$$ **(2)**

(ii) Find the value of A, giving your answer in terms of π. **(5)**

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4. $$f(x) = \frac{1}{\sqrt{1-9x^2}}$$

(*a*) Expand f(*x*) in ascending powers of *x* up to and including the term in x^4. **(3)**

(*b*) State the range of values of *x* for which the expansion is valid. **(1)**

(*c*) (i) Show that $\sqrt{\frac{1+3x}{1-3x}} = \frac{1+3x}{\sqrt{1-9x^2}}$. **(2)**

(ii) Hence obtain the expansion of $\sqrt{\frac{1+3x}{1-3x}}$, up to and including the term in x^5. **(2)**

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5. (*a*) The number *N* of bacteria in a culture is growing exponentially. The table shows values of *N* at different times *t*.

t	10	20	30	B
N	40	80	A	640

(3)

Find the values of A and B.

(*b*) A substance is decaying exponentially. After *t* years, its mass *m* grams is given by

$$m = 500\ e^{-0.1t}.$$

(i) Find the value of *m* when $t = 10$. **(1)**

(ii) Find the value of *t* when $m = 300$. **(3)**

(iii) Find the rate at which the mass is decreasing when $t = 20$. **(3)**

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6. (*a*) Find,

(i) $\int x \ln x \ \mathrm{d}x$ **(3)**

(ii) $\int \ln x \ \mathrm{d}x$ **(3)**

(*b*) By using a suitable substitution, show that

$$\int_1^{-2} x\sqrt{(x+3)}\ \mathrm{d}x = \frac{8}{5}.$$ **(6)**

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7. Petrol is poured into a container at a constant rate of 10 $\text{cm}^3\ \text{s}^{-1}$. After t seconds petrol is leaking from the container at a rate of $\frac{V}{4}\ \text{cm}^3\ \text{s}^{-1}$, where $V\text{cm}^3$ is the volume of petrol in the container at that time.

(a) Show that

$$-4\,\frac{\mathrm{d}V}{\mathrm{d}t} = V - 40.$$

(3)

Given that $V = 100$ when $t = 0$,

(b) Find a solution of the differential equation in the form $V = \mathrm{f}(t)$.

(7)

(c) Find the value which V approaches after a long time.

(1)

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8. The equations of the lines l and m are

$$\mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} 1 \\ -2 \\ -5 \end{pmatrix} + \mu \begin{pmatrix} a \\ b \\ 2 \end{pmatrix}$$

respectively. The lines l and m are perpendicular and they also intersect.

(a) Find the values of a and b.

(6)

(b) Find the position vector of the point of intersection.

(3)

(c) Calculate the acute angle between l and the line with equation $\mathbf{r} = \begin{pmatrix} -1 \\ 4 \\ 7 \end{pmatrix} + s \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$, giving your answer to the nearest degree.

(3)

END **TOTAL 75 MARKS**